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Article (Published Version)

Antunes, Nuno D, Copeland, Edmund J, Hindmarsh, Mark and Lukas, André (2003) Kinky Brane worlds. *Physical Review D*, 68 (6). 066005. ISSN 0556-2821

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Kinky brane worlds

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 (Received 27 January 2003; published 23 September 2003)

We present a toy model for five-dimensional heterotic M theory where bulk three-branes, originating in 11 dimensions from M five-branes, are modeled as kink solutions of a bulk scalar field theory. It is shown that the vacua of this defect model correspond to a class of topologically distinct M-theory compactifications. Topology change can then be analyzed by studying the time evolution of the defect model. In the context of a four-dimensional effective theory, we study in detail the simplest such process, that is, the time evolution of a kink and its collision with a boundary. We find that the kink is generically absorbed by the boundary thereby changing the boundary charge. This opens up the possibility of exploring the relation between more complicated defect configurations and the topology of brane-world models.

DOI: 10.1103/PhysRevD.68.066005

PACS number(s): 11.25.Mj, 11.27.+d, 98.80.Cq

I. INTRODUCTION

The single most important problem in trying to make contact between string or M theory and low-energy physics is probably the large number of degenerate and topologically distinct vacua of the theory. It is usually stated that nonperturbative effects will eventually lift most of this degeneracy. However, despite the advances over recent years in understanding nonperturbative string- and M theory there is very little indication of progress in this direction. In fact, with the advent of M theory and concepts such as branes and brane-world theories new classes of vacua have been constructed and, as a consequence, the degeneracy problem has perhaps grown even more serious. It seems worthwhile, therefore, to ask whether the cosmological evolution rather than inherent nonperturbative effects of the theory may play a prominent role in selecting the vacuum state. Indeed, it is known that the degeneracy of some vacua (particularly among those with a large number of supersymmetries) will not be lifted nonperturbatively, suggesting cosmology will have some role to play.

The first task to tackle, in this context, is the formulation of a workable theory capable of describing a number of topologically different vacua and transitions among them. As a second step, one will have to analyze the cosmological evolution of this theory. It is precisely these two problems which will be the main topic of the present paper.

The class of vacua we will use in our approach is provided by compactification of heterotic M theory [1] on Calabi-Yau three folds [2–5] resulting in five-dimensional brane-world theories [6–8]. These theories are defined on a space-time with two four-dimensional boundaries corresponding to the fixed planes of the orbifold S^1/Z_2 and, in addition, may contain bulk three-branes which originate from M five-branes wrapping two-cycles in the Calabi-Yau space [2,5]. The associated effective actions are five-dimensional gauged $N=1$ supergravity theories in the bulk

coupled to four-dimensional $N=1$ theories residing on the two boundaries and the three-branes. The prospects for particle-physics model building within this class of compactifications is quite promising and a number of models with attractive particle-physics properties on the “observable” boundary have been constructed [9–15]. The simplest way to characterize topologically different compactifications from the viewpoint of the five-dimensional effective theories is by using the charges α_1 and α_2 on the boundaries and the three-brane charge α_3 . These charges are not independent but must satisfy the cohomology constraint $\alpha_1 + \alpha_2 + \alpha_3 = 0$ which follows from anomaly cancellation. Two five-dimensional effective theories with different sets of charges $(\alpha_1, \alpha_2, \alpha_3)$ originate from topologically distinct compactifications. A transition between two such theories may occur through a small-instanton transition [16,17] when a three-brane collides with one of the boundaries. The three-brane can then be “absorbed” by the boundary and, correspondingly, the boundary charge is changed by the amount carried by the incoming three-brane. This change in the boundary charge indicates a more dramatic transition in the boundary theory. For example, the gauge group and the amount of chiral matter [18] may be altered as a consequence of the internal topology change.

The goal of this paper is to find a five-dimensional (toy) model which provides a unified description for the above class of topologically distinct vacua, in the simplest setting, and allows for transitions between them. While, for simplicity, we will assume that the topology of space-time both in the internal Calabi-Yau space and in the orbifold direction remains unchanged we will allow for transitions corresponding to a topology change in the internal gauge-field instantons on the boundaries and a change in the number and charges of three-branes. Our basic method will be, starting with five-dimensional heterotic M theory in its simplest form, to model the three-branes as topological defects (kinks) of a new bulk scalar field χ . A similar approach was used in [19], and in a more restrictive setting it was shown that D-branes may be described as kinks of a tachyon field [20,21]. We do not claim, of course, that our specific bulk scalar field model provides the correct definition of M theory in these backgrounds. In particular, it clearly fails to include the tensionless string which appears at the small instanton

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transition, as a consequence of the brane-boundary collision. We do show however that the defect model in the background of its various vacuum states reproduces the five-dimensional M-theory effective actions with different charges $(\alpha_1, \alpha_2, \alpha_3)$, corresponding to topologically distinct M-theory compactifications. All that said, the results of this work can still be approached independently of their hypothetical connection to M theory, as a study of kink evolution in brane-world models with boundaries.

Time-evolution of the defect model and the scalar χ in particular then allows for a transition between these topologically distinct configurations. We will study in detail the simplest such transition, namely the collision of a three-brane kink with one of the boundaries. This type of process has the added interest that it could be of fundamental cosmological importance, as it has been discussed extensively in the context of ekpyrotic universe models [22,23]. The collision will be studied by calculating the four-dimensional effective action for the defect model in the background of such a kink. As we will see from this four-dimensional action, the collision process indeed generically leads to an absorption of the kink and a change in the boundary charge by the amount carried by the kink. Hence, we have established the existence of one of the elementary topology-changing processes in our defect model. This opens up the possibility, subject to ongoing research, that a study of more complicated configurations, such as brane-networks, will provide insight into topological properties of brane-world models.

The plan of the paper is as follows. In the next section, we will introduce the five-dimensional effective actions from heterotic M theory, in their simplest form. For later reference, we will also review the associated four-dimensional effective theories. Section III then presents our defect model and explains how, precisely, it is related to the M-theory actions. In Sec. IV, we will compute the four-dimensional effective action for the defect model in the background of a kink and Sec. V presents the resulting evolution equations. Section VI is devoted to a detailed study of the kink evolution and its collision with a boundary, based on these equations. A conclusion and outlook is presented in Sec. VII.

II. EFFECTIVE ACTIONS FROM HETEROTIC M THEORY

To set the scene, we will now describe the five-dimensional brane world theories for which we would like to find a smooth defect-model. These brane-world theories can be viewed as a minimal version of five-dimensional heterotic M theory [6]. For later purposes, it will also be useful to review the four-dimensional effective action associated to these brane-world theories.

Coordinates for the five-dimensional space M_5 are denoted by x^α where $\alpha, \beta, \dots = 0, 1, 2, 3, 5$. We also introduce four-dimensional indices $\mu, \nu, \dots = 0, 1, 2, 3$. The coordinate $y \equiv x^5$ is compactified on an orbi-circle S^1/Z_2 in the usual way, that is, by first compactifying y on a circle with radius ρ and then dividing by the Z_2 orbifold action $y \rightarrow -y$. Taking the y coordinate in the range $y \in [-\pi\rho, \pi\rho]$ with the end points being identified the two resulting four-dimensional

fixed planes (boundaries), denoted by M_4^1 and M_4^2 , are located at $y=0$ and $y=\pi\rho$, respectively. Such a geometry is obtained by compactifying 11-dimensional heterotic M theory on a Calabi-Yau space. If the five-branes present in the 11-dimensional theory are included in this compactification they lead, upon wrapping a two-cycle in the Calabi-Yau space, to bulk three-branes in the five-dimensional brane-world theories. For simplicity, we will consider a single such three-brane whose world-volume we denote by M_4^3 . We also need to include the Z_2 mirror of this three-brane with world-volume \tilde{M}_4^3 . Three-brane world-volume coordinates are denoted by σ^μ . In the minimal version of the model the bulk fields consist of the metric and the dilaton Φ while the three-brane world-volume fields are simply the embedding coordinates $X^\alpha = X^\alpha(\sigma^\mu)$. The effective action for these fields is then given by [24]

$$S_5 = -\frac{1}{2\kappa_5^2} \left\{ \int_{M_5} \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{4} \partial_\alpha \Phi \partial^\alpha \Phi + \frac{1}{3} \alpha^2 e^{-2\Phi} \right] \right. \\ \left. + \int_{M_4^1} \sqrt{-g} 2\alpha_1 e^{-\Phi} + \int_{M_4^2} \sqrt{-g} 2\alpha_2 e^{-\Phi} \right. \\ \left. + \int_{M_4^3 \cup \tilde{M}_4^3} \sqrt{-g} |\alpha_3| e^{-\Phi} \right\}. \quad (2.1)$$

Note, that the dilaton Φ measures the size of the internal Calabi-Yau space which is, more precisely, given by $v e^\Phi$, where v is a fixed reference volume. It relates the five-dimensional Newton constant κ_5 to its 11-dimensional counterpart κ via

$$\kappa_5^2 = \frac{\kappa^2}{v}. \quad (2.2)$$

Further, α_i , where $i=1,2,3$, are the charges on the orbifold planes and the three-brane, respectively. They are quantized and can be written as integer multiples

$$\alpha_i = \sigma \beta_i, \quad \beta_i \in \mathbf{Z} \quad (2.3)$$

of the unit charge σ defined by

$$\sigma = \frac{\epsilon_0}{\pi\rho}, \quad \epsilon_0 = \left(\frac{\kappa}{4\pi} \right)^{2/3} \frac{2\pi^2\rho}{v^{2/3}}. \quad (2.4)$$

These charges satisfy the important cohomology condition

$$\sum_{i=1}^3 \alpha_i = 0 \quad (2.5)$$

which follows from anomaly cancellation in the 11-dimensional theory. The quantity α which appears in the above bulk potential is a sum of step-functions given by

$$\alpha = \alpha_1 \theta(M_4^1) + \alpha_2 \theta(M_4^2) + \alpha_3 [\theta(M_4^3) + \theta(\tilde{M}_4^3)]. \quad (2.6)$$

Finally, the induced metric $\gamma_{\mu\nu}$ on the three-brane world-volume is defined as the pull-back

$$\gamma_{\mu\nu} = \partial_\mu X^\alpha \partial_\nu X^\beta g_{\alpha\beta} \quad (2.7)$$

of the space-time metric.

For positive three-brane charge, $\alpha_3 > 0$, the above action can be embedded into a five-dimensional $N=1$ bulk supergravity theory coupled to four-dimensional $N=1$ theories on the boundaries and the branes. The details of this supergravity theory have been worked out in Ref. [24]. In the case of an anti-three-brane, that is for $\alpha_3 < 0$, while bulk supersymmetry is preserved everywhere locally, it is broken globally. Technically, this happens because the chirality of the four-dimensional supersymmetry preserved on the three-brane is opposite to the one on the orbifold fixed planes. Such non-supersymmetric heterotic models containing antibranes have not been studied in much detail, so far. We have included this possibility here because it will naturally arise later in our discussion of the defect model. The generalization to include more than one three-brane is straightforward. It simply amounts to replacing the Nambu-Goto type three-brane action in the third line of Eq. (2.1) by a sum over such actions (with generally different three-brane charges) and modifying the cohomology condition (2.5) and the definition of α , Eq. (2.6), accordingly.

Note that two actions of the type (2.1) but with different sets of charges α_i correspond to topologically different M-theory compactifications. Specifically, the charges α_1 and α_2 on the boundaries are related to gravitational and gauge instanton numbers. If we keep the topology of the Calabi-Yau space fixed, as discussed, different values of α_1 and α_2 indicate a different topology of the internal gauge bundles. As a consequence, the values of α_1 , α_2 are also correlated with other properties of the boundary theories, such as the types of gauge groups and the amount of chiral matter. Different values of α_3 imply different internal wrapping numbers for the five-branes and, hence, clearly indicate different topologies.

For the case of a three-brane (rather than an anti-three-brane), the action (2.1) has a BPS domain-wall vacuum [6,24] given by

$$ds^2 = a_0^2 h dx^\mu dx^\nu \eta_{\mu\nu} + b_0^2 h^4 dy^2 \quad (2.8)$$

$$e^\Phi = b_0 h^3 \quad (2.9)$$

$$X^\mu = \sigma^\mu \quad (2.10)$$

$$X^5 = Y = \text{const.} \quad (2.11)$$

Here the function $h = h(y)$ is defined by

$$h(y) = -\frac{2}{3} \begin{cases} \alpha_1 |y| + c_0 & \text{for } 0 \leq |y| \leq Y \\ (\alpha_1 + \alpha_3) |y| - \alpha_3 Y + c_0 & \text{for } Y \leq |y| \leq \pi \rho \end{cases} \quad (2.12)$$

and a_0 , b_0 and c_0 are constants. Note that this solution is not smooth across the three-brane reflecting the fact that the three-brane as described by Eq. (2.1) is infinitely thin. Such a static BPS solution does not exist for the anti-three-brane since the sum of the tensions $\alpha_1 + \alpha_2 + |\alpha_3|$ does not vanish for $\alpha_3 < 0$ by virtue of the cohomology condition (2.5). In fact, solutions which couple to an anti-three-brane will, in general, be time-dependent.

For later reference, it will be useful to discuss the four-dimensional effective action associated to the brane-world model (2.1) and the above BPS vacuum. It is given by [25,24]

$$S_4 = -\frac{1}{2\kappa_P^2} \int_{M_4} \sqrt{-g_4} \left[\frac{1}{2} R_4 + \frac{1}{4} \partial_\mu \phi \partial^\mu \phi + \frac{3}{4} \partial_\mu \beta \partial^\mu \beta + \frac{q_3}{2} e^{\beta - \phi} \partial_\mu z \partial^\mu z \right]. \quad (2.13)$$

The three scalar fields ϕ , β and z have straightforward interpretations in terms of the underlying higher-dimensional theories. The field ϕ , as the zero mode of the five-dimensional scalar Φ , specifies the volume of the internal Calabi-Yau space averaged over the orbifold. More precisely,

this average volume is given by $v e^\phi$. The scalar β , on the other hand, originates from the (55)-component of the five-dimensional metric and measures the size $\pi \rho e^\beta$ of the orbifold. Finally, z represents the position of the three-brane and is normalized to be in the range $z \in [0, 1]$ with the end points corresponding to the two boundaries of five-dimensional space-time. The four-dimensional Newton constant κ_P is related to its five-dimensional cousin by

$$\kappa_P^2 = \frac{\kappa_5^2}{2\pi\rho}. \quad (2.14)$$

Finally, the three-brane charge

$$q_3 = \pi \rho \alpha_3 = \epsilon_0 \beta_3, \quad \beta_3 \in \mathbf{Z} \quad (2.15)$$

is quantized in units of ϵ_0 as defined in Eq. (2.4) and is positive for the case under discussion.

As expected, the action (2.13) can be obtained from an $N=1$ supergravity theory by a suitable truncation. The Kahler potential for this supergravity theory has been first given in Ref. [25]. An important quantity which governs the validity of the effective action (2.13) is the strong-coupling expansion parameter

$$\epsilon = \epsilon_0 e^{\beta - \phi}. \quad (2.16)$$

It measures the relative size of string loop corrections to the four-dimensional action (2.13) or, equivalently, the strength of the warping in the orbifold direction from a five-dimensional viewpoint. The effective action (2.13) is valid as long as $\epsilon < 1$ and can be expected to break down otherwise. Another reason for a breakdown of the four- as well as the five-dimensional effective theory is the five-brane approaching one of the boundaries, that is, $z \rightarrow 0$ or $z \rightarrow 1$. In this case, the underlying heterotic M theory may undergo a small-instanton transition [16,17] which leads to the M five-brane being converted into a gauge-field instanton (or, so called gauge five-brane [26]) on the boundary. In such a process, properties of the boundary theory, such as the gauge group and the amount of chiral matter, can change dramatically as a result of the internal topology change [18]. In our simple five-dimensional model (2.1) such a modification of the boundary theory is indicated by a change in the boundary charge α_1 or α_2 by the amount of incoming five-brane charge. It is clear, however, that the actions (2.1) or (2.13) are not capable of describing such a jump in the boundary charge in a dynamical way. In fact, the four-dimensional action (2.13) does not retain any memory of the presence of the boundaries as $z \rightarrow 0,1$. This can also be seen from the moving-brane solutions to Eq. (2.13) found in Ref. [27] and will be explained in more detail later. As we will see, our defect model, to be presented in the next section, will considerably improve on these points.

III. MODELING HETEROTIC BRANE-WORLD THEORIES

We would now like to find a “smooth” model, replacing the five-dimensional action (2.1), where the three-brane is not put in “by hand” but, rather, obtained as defect solution to the theory. Such a model should have, as a solution, a smooth version of the BPS domain wall (2.8)–(2.12). Note, that we will not attempt to find a smooth description for the orbifold fixed planes. Their nature, as part of the space-time geometry, is entirely different from the one of the three-branes. In particular, the fixed plane tensions α_1, α_2 can be negative whereas the three-brane tension $|\alpha_3|$ is always positive.

Modeling codimension-one objects such as our three-branes is usually achieved using kink-solutions of scalar field theories [19]. This is indeed what we will do here. We, therefore, supplement the bulk field content of the five-dimensional theory by a second scalar field χ . For this bulk scalar along with the dilaton Φ and the five-dimensional metric, we propose the following action:

$$\begin{aligned} \tilde{S}_5 = & -\frac{1}{2\kappa_5^2} \left\{ \int_{M_5} \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{4} \partial_\alpha \Phi \partial^\alpha \Phi \right. \right. \\ & + \frac{1}{2} e^{-\Phi} \partial_\alpha \chi \partial^\alpha \chi + V(\Phi, \chi) \Big] + \int_{M_4} \sqrt{-g} 2W \\ & \left. \left. - \int_{M_4^2} \sqrt{-g} 2W \right\}. \end{aligned} \quad (3.1)$$

We require that the potential V be obtained from a “superpotential” W following the general formula [28]

$$V = \frac{1}{2} G^{IJ} \partial_I W \partial_J W - \frac{2}{3} W^2, \quad (3.2)$$

where G_{IJ} is the sigma-model metric and indices I, J, \dots label the various scalar fields Φ^I . For our specific action (3.1), we have two scalar fields $(\Phi^I) = (\Phi, \chi)$ and the sigma-model metric is explicitly given by

$$G = \text{diag} \left(\frac{1}{2}, e^{-\Phi} \right). \quad (3.3)$$

Further, we propose the following form for the superpotential:

$$W = e^{-\Phi} w(\chi) \quad (3.4)$$

where w is an, as yet, unspecified function of χ . Using the general expression (3.2) this results in a potential

$$V = \frac{1}{3} e^{-2\Phi} w^2 + \frac{1}{2} e^{-\Phi} U, \quad U = \left(\frac{dw}{d\chi} \right)^2. \quad (3.5)$$

Note that, in Eq. (3.1), we have omitted the Nambu-Goto type action for the three-brane corresponding to the third line of the M-theory effective action (2.1). The reason is, of course, that we would like to recover the three-brane as a kink-solution of the new scalar field χ . For this to work out, the potential U has to have a nontrivial vacuum structure. In fact, since the original three-brane charge is an (arbitrary) integer multiple of a certain unit, we need an infinite number of equally spaced minima. More precisely, we require that the potential U satisfies the following properties:

U is periodic with period v , that is $U(\chi + v) = U(\chi)$.

U has minima at $\chi = \chi_n = nv$ for all $n \in \mathbf{Z}$.

U vanishes at the minima, that is $U(\chi_n) = 0$.

These requirements can be easily translated into conditions on the function w which determines the superpotential. Clearly, from the second and third condition, the derivative of w has to vanish at all minima $\chi_n = nv$ of U . The definition (3.5) of w in terms of U involves a sign ambiguity which allows one, using the first condition on U above, to make w periodic as well. However, the structure of the action (3.1) makes it clear that the “vacuum values” $w(\chi_n)$ of w have to reproduce the charges on the orbifold planes. We, therefore, define w as

$$w(\chi) = \int_0^\chi d\tilde{\chi} \sqrt{U(\tilde{\chi})} \quad (3.6)$$

which implies quasiperiodicity, that is,

$$w(\chi + v) = w(\chi) + w(v). \quad (3.7)$$

We have plotted the typical form of U and w in Fig. 1.

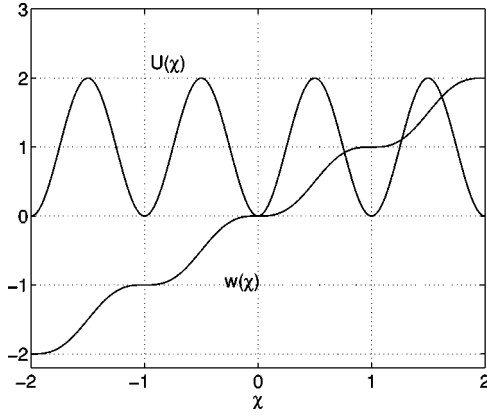


FIG. 1. Shown is the typical shape of the superpotential w and the potential U (in units of σ) as a function of the scalar field χ (in units of v).

For much of our discussion the concrete form of the potential will be irrelevant as long as the above conditions are met. A specific example, however, is provided by the sine-Gordon potential

$$U = m^2 \left[1 - \cos\left(\frac{2\pi\chi}{v}\right) \right], \quad (3.8)$$

where m is a constant. The associated superpotential w is easily obtained by integration.

This concludes the setup of our model. Let us now discuss how, precisely, this model corresponds to the brane-world theory (2.1) introduced earlier. The simplest solution for χ is to be in one of its vacuum states, that is, $\chi = \chi_n = nv$ for some integer n , throughout space-time. In this case, the superpotential and potential reduce to

$$W = e^{-\Phi} w(\chi_n), \quad V = \frac{1}{3} e^{-2\Phi} w(\chi_n)^2. \quad (3.9)$$

Substituting this back into the action (3.1) and comparing with the M-theory result (2.1) shows that this precisely corresponds to a situation without a bulk three-brane. In particular, one concludes that the boundary charge α_1 has to be identified with the value $w(\chi_n) = w(nv) = nw(v)$ of the superpotential at the respective minimum.¹ This is, of course, the more precise reason why we have required the superpotential to be quasiperiodic rather than periodic. Furthermore, we learn that the elementary unit of charge σ in the M-theory model [see Eq. (2.4)] corresponds to $w(v)$, that is,

$$\sigma = w(v) = \int_0^v d\xi \sqrt{U(\chi)}. \quad (3.10)$$

The next more complicated solutions are static kinks where the scalar field χ interpolates between two of its

minima as one moves along the orbifold direction. Due to the cross-couplings in the action (3.1) also the dilaton Φ and the metric necessarily have a nontrivial profile in this case. To find such solutions, an appropriate *Ansatz* is provided by

$$ds^2 = e^{2A(y)} dx^\mu dx^\nu \eta_{\mu\nu} + e^{2B(y)} dy^2 \quad (3.11)$$

$$\Phi = \Phi(y) \quad (3.12)$$

$$\chi = \chi(y). \quad (3.13)$$

The four y -dependent functions A , B , Φ , χ are subject to the second order bulk equations of motion to be derived from the first line in Eq. (3.1) and the boundary conditions

$$e^{-B} A' = -\frac{1}{3} W = -\frac{1}{3} e^{-\Phi} w, \quad (3.14)$$

$$e^{-B} \Phi' = 2 \frac{\partial W}{\partial \Phi} = -2 e^{-\Phi} w, \quad (3.15)$$

$$e^{-B} \chi' = e^{\Phi} \frac{\partial W}{\partial \chi} = \frac{dw}{d\chi}. \quad (3.16)$$

Here, the prime denotes the derivative with respect to y and the equations hold at both boundaries, that is, at $y=0$ and $y=\pi\rho$. The first equality in each equation is easily derived from Eq. (3.1) including the boundary terms while the second one follows from inserting the explicit form of the superpotential (3.4).

Instead of dealing with the second order equations to obtain explicit solutions it is much simpler to consider the first order BPS-type equations. Their existence is guaranteed by the special form of our scalar field potential V as being obtained from a superpotential [28]. Concretely, inserting the *Ansätze* (3.11)–(3.13) into the bulk part of the action (3.1) one obtains an energy functional

$$\begin{aligned} E &\sim \int dy e^{4A} \left[-6 e^{-2B} A'^2 + \frac{1}{4} e^{-2B} \Phi'^2 \right. \\ &\quad \left. + \frac{1}{2} e^{-\Phi-2B} \chi'^2 + V \right], \\ &= \int dy e^{4A} \left[\frac{1}{4} \left(e^{-B} \Phi' \mp 2 \frac{\partial W}{\partial \Phi} \right)^2 \right. \\ &\quad \left. + \frac{1}{2} e^{-\Phi} \left(e^{-B} \chi' \mp e^{\Phi} \frac{\partial W}{\partial \chi} \right)^2 \right. \\ &\quad \left. - \frac{2}{3} (3 e^{-B} A' \pm W)^2 \right] \pm [e^{4\alpha} W]_{y=0}^{y=\pi\rho} \end{aligned} \quad (3.17)$$

which can be written in Bogomol'nyi perfect square form. This leads to the following first order equations:

$$e^{-B} A' = \mp \frac{1}{3} W = \mp \frac{1}{3} e^{-\Phi} w, \quad (3.18)$$

¹Note that, in the absence of three-branes, we have $\alpha_2 = -\alpha_1$ from the cohomology condition (2.5). Therefore, also the charge on the second boundary is correctly being taken care of by our model.

$$e^{-B}\Phi' = \pm 2 \frac{\partial W}{\partial \Phi} = \mp 2e^{-\Phi}w, \quad (3.19)$$

$$e^{-B}\chi' = \pm e^{\Phi} \frac{\partial W}{\partial \chi} = \pm \frac{dw}{d\chi}. \quad (3.20)$$

Again, the second equality in each line follows from inserting the explicit superpotential (3.4). The scale factor B is, of course, a gauge degree of freedom and can, for example, be set to a constant. It is clear then that Eq. (3.20) for χ decouples from the other two. This χ equation is, in fact, exactly the same first order equation one would derive for a single scalar field χ with potential U in a flat background. It is, therefore, clear and can be seen by direct integration, that this equation admits kink solutions where χ interpolates between a certain minimum $\chi = \chi_n = nv$ of U at $y \rightarrow -\infty$ and one of its neighboring minima at $y \rightarrow +\infty$. More precisely, for the choice of the upper (lower) sign in Eq. (3.20) the minimum at $\chi = (n+1)v$ [$\chi = (n-1)v$] is approached for $y \rightarrow +\infty$. The corresponding solutions for A and Φ can then be obtained by inserting this kink solution and integrating Eqs. (3.18) and (3.19). In the next section, this will be carried out in a more precise way. In addition, the solutions obtained in this way have to satisfy the boundary conditions (3.14)–(3.16). Clearly, this is automatically the case if the upper sign in the first order equations (3.18)–(3.20) has been chosen, that is, if the kink interpolates between the minima $\chi = nv$ and $\chi = (n+1)v$ for increasing y . For the lower sign, on the other hand, there is no chance to satisfy the boundary conditions and, hence, no solutions of the type considered here exist in this case. The interpretation of these results is straightforward. While both types of kinks are on the same footing as far as the bulk equations are concerned the boundary conditions distinguish what should then be called an antikink, interpolating between $\chi = nv$ and $\chi = (n-1)v$, from a kink, interpolating between $\chi = nv$ and $\chi = (n+1)v$. While the latter represents a BPS solution of the theory, the former carries the wrong orientation to be compatible with the boundaries and, in fact, will only exist as a dynamical object. This is in direct analogy with the properties of three-branes and anti-three-branes in our original M-theory model (2.1). The symmetry between kinks and antikinks was broken by the choice of sign of the boundary terms in Eq. (3.1). A BPS antikink solution can be obtained by swapping the signs, a configuration which would then model branes with charges satisfying $\alpha_1 - \alpha_2 = +1$. We should also stress that although the bulk first-order equations for the kink coincide with the boundary conditions, this makes the effects of the boundary in no way “trivial” as far as the kink solution is concerned. For example, a Lorentz boosted kink configuration is a solution of the bulk equations of motion but it does not obey the boundary conditions. A kink moving towards one of the boundaries will necessarily feel its presence, its motion clearly differing from the free-boundary situation.

For the case of a kink, we would like to make the correspondence with the M-theory model more precise. Let us consider a kink solution to Eqs. (3.18)–(3.20) and (3.14)–(3.16) with the kink width being small (compared to the size

of the orbifold) and the core of the kink sufficiently away from the boundaries. In this case, the profile for χ and $w(\chi)$ can be approximated by a step-function. Specifically, we have $w(\chi) \approx nw(v)$ to the left of the kink and $w(\chi) \approx (n+1)w(v)$ to the right. Inserting this into Eqs. (3.18), (3.19) and the boundary conditions (3.14), (3.15) for A and Φ and solving the resulting system precisely leads to the BPS three-brane solution given by Eqs. (2.8), (2.9), and (2.12). The charges α_i appearing in this solution are given by

$$\alpha_1 = n\sigma, \quad \alpha_2 = -(n+1)\sigma, \quad \alpha_3 = \sigma, \quad (3.21)$$

where we have used our earlier identification (3.10) of the superpotential value $w(v)$ with the elementary charge unit σ . Hence, our model allows for a solution which can be interpreted as a smooth version of the M-theory domain wall coupled to a single-charged three-brane.

More generally, we would like to discuss the relation between the action (3.1) in the background of a kink solution and the M-theory action (2.1). To do this, we should allow for fluctuations of the kink. It is well-known [29] that, for sufficiently small width, the hypersurface prescribed by the kink’s core is a minimal surface and is, therefore, adequately described by a Nambu-Goto action. Practically, this implies that the kinetic term for χ and the U potential term in the action (3.1) can be effectively replaced by a Nambu-Goto action describing the dynamics of the core of the kink. Of course, this core has to be identified with the three-brane in the M-theory model. It is easy to show that, by virtue of Eq. (3.10), the tension in this effective Nambu-Goto action is given by σ which is the correct value for a single-charged three-brane with $\beta_3 = 1$. Further, the superpotential w in such a kink background can be effectively replaced by a step-function, as discussed above. Using the identification (3.21) of charges, it is easy to see that the superpotential w precisely equals the function α , defined in Eq. (2.6), in this limit. As a consequence, the second potential term in Eq. (3.1) proportional to $e^{-2\Phi}w^2$ precisely reproduces the bulk potential in the M-theory action (2.1). Similarly, the boundary potentials in Eq. (3.1) match the boundary potentials in Eq. (2.1) using that $w[\chi(y=0)] \approx n\sigma = \alpha_1$ and $w[\chi(y=\pi\rho)] \approx (n+1)\sigma = -\alpha_2$. Although there are no BPS antikink solutions, it is clear that a similar argument can be made for the action (3.1) in the background of an antikink leading to the M-theory action (2.1) with an anti-three-brane.

In summary, we have seen that the action (3.1) in the background of various vacuum configurations of the field χ reproduces different versions of the M-theory effective action (2.1). For a constant field χ located in one of the minima of U , we have reproduced the M-theory action without three-branes. For a kink (antikink) background with sufficiently small width away from the boundaries we have obtained the M-theory action with a single-charged three-brane (anti-three-brane). Note that, while from the viewpoint of the smooth model (3.1) these cases merely correspond to different configurations of the field χ , they represent different effective actions on the M-theory side. As we have discussed, these different effective actions arise from topologically distinct compactifications of the 11-dimensional M theory.

While these compactifications are known to be related by topology-changing transitions such as small-instanton transitions these processes cannot be described by the action (2.1). What we have seen is, that our smooth defect model incorporates a number of these topologically distinct configurations within a single theory and may describe transitions between them as the scalar field χ evolves in time. In the subsequent sections, we will study the simplest example for such a transition, namely the collision of a kink with one of the boundaries.

A final comment concerns the question of multicharged branes. Clearly, multicharged BPS three-branes with $\beta_3 > 1$ are allowed in the M-theory model (2.1). However, our defect model (3.1) does not have exact BPS multi-kink solutions as long as the potential U is smooth at its minima. The reason is that, for smooth U , a kink solution does not reach a minimum within a finite distance, as can be easily seen from Eq. (3.20) with U expanded around a minimum. As a consequence, single-kink solutions cannot be “stacked” to produce exact multikink solutions. There are a number of options available to remove this apparent discrepancy. Firstly, the model (3.1) as it stands does have approximate multikink solutions (with exponential accuracy) which could be identified with multicharged three-branes. Secondly, if the potential U is continuous but nonsmooth at its minima a kink solution can reach a minimum within a finite distance. There is no obstruction then to build up exact multikinks by stacking single-kink solutions. Thirdly, some multiscalar field models are known to admit multikink solutions [30]. So, we may generalize the action (3.1) by adding more than one scalar field. For the purpose of this paper, we will not implement any of these possibilities explicitly but, rather, focus on single-kink solutions in the following.

IV. THE FOUR-DIMENSIONAL EFFECTIVE ACTION OF A KINK SOLUTION

We would now like to study one of the simplest dynamical processes in the context of our defect model, namely the time-evolution of a kink solution and its collision with a boundary. For a sufficiently slow evolution this can be conveniently studied in the context of the four-dimensional effective theory associated to Eq. (3.1) in the presence of a kink. The purpose of this section is to compute this effective four-dimensional theory. As we will see, this computation can be pushed a long way without specifying an explicit potential U . We will, therefore, keep U general throughout this section. An explicit example for U will be studied in the next section.

Our first step is to write the kink solution in a form which makes the dependence on the various integration constants (which will be promoted to four-dimensional moduli fields later on) as explicit as possible. We find that the kink solution to Eqs. (3.18)–(3.20) and Eqs. (3.14)–(3.16) interpolating between the minima $\chi = \chi_n = nv$ and $\chi = \chi_{n+1} = (n+1)v$ for increasing y can be cast in the form

$$\chi = C[e^{\beta}\mu^{-1}(y/\pi\rho - z)], \quad (4.1)$$

$$e^{\Phi} = e^{\phi}[1 + \epsilon_0 e^{\beta-\phi} f(y, \beta, z)], \quad (4.2)$$

$$A = A_0 + \frac{1}{6}\Phi, \quad (4.3)$$

$$B = \beta, \quad (4.4)$$

where we recall that A and B are the scale factors in the five-dimensional metric as defined in Eqs. (3.11)–(3.13). The functions C and f in the above solution can be expressed in terms of the potential as follows:

$$C^{-1}(\chi) = \frac{1}{\pi\rho\mu} \int_{[n+(1/2)v]v}^{\chi} \frac{d\tilde{\chi}}{\sqrt{U(\tilde{\chi})}}, \quad (4.5)$$

$$f(y, \beta, z) = -\frac{2}{\pi\rho\epsilon_0} \times \int_{y_0}^y d\tilde{y} w \{ C[e^{\beta}\mu^{-1}(\tilde{y}/\pi\rho - z)] \}. \quad (4.6)$$

Here ϕ , β , z , A_0 and y_0 are integration constants, while μ is a constant which measures the width of the kink in units of $\pi\rho$. It is clear from the form of the metric (3.11) that the constant A_0 can be absorbed into the four-dimensional metric. As we will see, it is, however, convenient to keep this constant explicitly since it can be used to canonically normalize the four-dimensional Einstein-Hilbert term. For our subsequent discussion, let us define the average $\langle h \rangle$ of a function $h = h(y)$ over the orbifold by

$$\langle h \rangle = \frac{1}{\pi\rho} \int_0^{\pi\rho} dy h(y). \quad (4.7)$$

Since the constants y_0 and ϕ really describe the same degree of freedom, we can fix y_0 by requiring that $\langle f \rangle = 0$. With this convention, the integration constant ϕ has a clear geometrical interpretation, namely e^{ϕ} represents the orbifold average of the dilaton e^{Φ} . Similarly, e^{β} measures the orbifold size in units of $\pi\rho$. The final integration constant z specifies the position of the kink's core [the position where $\chi = (n + \frac{1}{2})v$] in the orbifold direction. Values $z \in [0, 1]$ imply that the kink's core is located within the boundaries of five-dimensional space and is, hence, physically present. Further, $z \rightarrow 0, 1$ indicates collision of the kink with one of the boundaries. For $z \notin [0, 1]$ the core is outside the physical region and we can merely think of z as the virtual position of the core were space-time to continue beyond the boundaries. In this case, the physical part of the kink, located between the boundaries, is only its tail. In the limiting case $z \rightarrow \pm\infty$ the kink disappears completely and we approach one of the trivial vacuum states of the theory with either $\chi = nv$ or $\chi = (n+1)v$ throughout five-dimensional space-time depending on whether $z \rightarrow +\infty$ or $z \rightarrow -\infty$. Also note that the function C , defined in Eq. (4.5), is independent of all integration constants and can be computed for a given potential U .

We should now promote all integration constants in our kink solution to four-dimensional moduli fields. This leads to

three scalar fields $(\phi^I) = (\phi, \beta, z)$ and the four-dimensional effective metric $g_{4\mu\nu}$. Accordingly, the *Ansätze* (3.11)–(3.13) should then be modified to

$$ds^2 = e^{2A(y, \phi^I)} dx^\mu dx^\nu g_{4\mu\nu} + e^{2B(y, \phi^I)} dy^2, \quad (4.8)$$

$$\Phi = \Phi(y, \phi^I), \quad (4.9)$$

$$\chi = \chi(y, \phi^I), \quad (4.10)$$

where A , B , Φ and χ are as in Eqs. (4.1)–(4.4) but with $(\phi^I) = (\phi, \beta, z)$ now viewed as functions of the external coordinates x^μ .

We are now ready to compute the four-dimensional effective action. Inserting the *Ansätze* (4.8)–(4.10) into the action (3.1) and integrating over the orbifold direction we obtain the following result:

$$\tilde{S}_4 = -\frac{1}{2\kappa_P^2} \int_{M_4} \sqrt{-g_4} \left[\frac{1}{2} R_4 + \frac{1}{2} G_{IJ} \partial_\mu \phi^I \partial^\mu \phi^J \right]. \quad (4.11)$$

The sigma-model metric G_{IJ} is given by

$$G_{IJ} = 2 \left\langle e^{2A+B} \left[-3 \partial_I A \partial_J A - 3 \partial_{(I} A \partial_{J)} B + \frac{1}{4} \partial_I \Phi \partial_J \Phi + \frac{1}{2} e^{-\Phi} \partial_I \chi \partial_J \chi \right] \right\rangle, \quad (4.12)$$

where $\partial_I = \partial / \partial \phi^I$ and $(\phi^I) = (\phi, \beta, z)$. Further, in order to obtain an Einstein-frame action we have required that

$$\langle e^{2A+B} \rangle = 1. \quad (4.13)$$

This indeed fixes the constant A_0 in Eq. (4.3) to be

$$e^{2A_0} = e^{-\beta} \langle e^{\Phi/3} \rangle^{-1}. \quad (4.14)$$

The four-dimensional Planck scale κ_P is defined by

$$\kappa_P^2 = \frac{\kappa_5^2}{2\pi\rho}, \quad (4.15)$$

as usual.

The remaining task is now to evaluate the expression (4.12) for the moduli-space metric using the kink solution (4.1)–(4.4). This leads to fairly complicated results, in general. There is, however, an approximation suggested by the original M-theory model which simplifies matters considerably. As discussed, the effective actions for heterotic M theory in Sec. II are valid only if the strong-coupling expansion parameter

$$\epsilon = \epsilon_0 e^{\beta - \phi} \quad (4.16)$$

is smaller than one. We are, therefore, led to compute the moduli-space metric (4.12) in precisely this limit which corresponds to small warping in the orbifold direction. Concretely, we will keep terms up to $\mathcal{O}(\epsilon)$ and neglect all terms of $\mathcal{O}(\epsilon^2)$ and higher in our computation. This implies a dra-

matic simplification since the function f , which enters the kink solution Eq. (4.2) with an $\mathcal{O}(\epsilon)$ suppression, drops out at this order. Inserting Eqs. (4.2)–(4.4) and (4.14) into Eq. (4.12), one then finds for the moduli-space metric

$$G = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{3}{2} + e^{-\phi} \langle (\partial_\beta \chi)^2 \rangle & e^{-\phi} \langle \partial_\beta \chi \partial_z \chi \rangle \\ 0 & e^{-\phi} \langle \partial_\beta \chi \partial_z \chi \rangle & e^{-\phi} \langle (\partial_z \chi)^2 \rangle \end{pmatrix} + \mathcal{O}(\epsilon^2). \quad (4.17)$$

Using the solution (4.1) for χ we finally obtain

$$G_{\phi\phi} = \frac{1}{2}, \quad (4.18)$$

$$G_{\beta\beta} = \frac{3}{2} + (e^{-\beta} \mu)^2 \epsilon_0 e^{\beta - \phi} \{ J_2 [e^\beta \mu^{-1} (1 - z)] - J_2 (-e^\beta \mu^{-1} z) \}, \quad (4.19)$$

$$G_{\beta z} = -e^{-\beta} \mu \epsilon_0 e^{\beta - \phi} \{ J_1 [e^\beta \mu^{-1} (1 - z)] - J_1 (-e^\beta \mu^{-1} z) \}, \quad (4.20)$$

$$G_{zz} = \epsilon_0 e^{\beta - \phi} \{ J_0 [e^\beta \mu^{-1} (1 - z)] - J_0 (-e^\beta \mu^{-1} z) \}, \quad (4.21)$$

as the only nonvanishing components of G . Here, the functions J_n are defined by

$$J_n(x) = \frac{(\pi\rho)^2 \mu}{\epsilon_0} \int_0^x d\tilde{x} \tilde{x}^n U[C(\tilde{x})] = \frac{1}{\sigma} \int_{C(0)}^{C(x)} d\chi [C^{-1}(\chi)]^n w'(\chi), \quad (4.22)$$

where we recall that the function C , defined in Eq. (4.5), can be computed for any given potential U and is, by itself, independent of the moduli. The above result, good to $\mathcal{O}(\epsilon)$, for the sigma model metric explicitly displays the complete moduli dependence of G and its only implicit features are the dependence on the potential U and a simple integral thereof. We find it quite remarkable that the calculation can be pushed this far without an explicit choice for the potential U .

The results (4.18)–(4.21) suggest the existence of another expansion parameter besides ϵ , namely the quantity $e^{-\beta} \mu$. It represents the ratio of the kink's width and the size of the orbifold. Working in a thin-wall approximation where this ratio is much smaller than one our results simplify even further. Clearly, we then have to good accuracy

$$G_{\beta\beta} = \frac{3}{2}, \quad G_{\beta z} = 0. \quad (4.23)$$

For the remaining nontrivial component G_{zz} we can explicitly carry out the integral (4.22) and find by inserting into Eq. (4.21)

$$G_{zz} = \epsilon_0 e^{\beta - \phi} F(\beta, z) \quad (4.24)$$

where

$$F(\beta, z) = \frac{1}{\sigma} (w\{C[e^{\beta} \mu^{-1}(1-z)]\} - w[C(-e^{\beta} \mu^{-1}z)]), \quad (4.25)$$

$$= \frac{1}{\sigma} [w(y = \pi\rho) - w(y = 0)]. \quad (4.26)$$

Here, the notation $w(y=0)$ [$w(y=\pi\rho)$] indicates the value of the superpotential evaluated for the kink solution at the boundary $y=0$ ($y=\pi\rho$).

To summarize, in the limit of both the strong-coupling expansion parameter and the ratio of wall to orbifold size being smaller than one, that is,

$$\epsilon = \epsilon_0 e^{\beta - \phi} < 1, \quad \frac{\mu}{e^{\beta}} < 1, \quad (4.27)$$

the moduli-space metric for the kink solution is well-approximated by

$$G = \text{diag}\left(\frac{1}{2}, \frac{3}{2}, \epsilon_0 e^{\beta - \phi} F(\beta, z)\right) \quad (4.28)$$

with associated four-dimensional effective action

$$\begin{aligned} \tilde{S}_4 = & -\frac{1}{2\kappa_p^2} \int_{M_4} \sqrt{-g_4} \left[\frac{1}{2} R_4 + \frac{1}{4} \partial_\mu \phi \partial^\mu \phi + \frac{3}{4} \partial_\mu \beta \partial^\mu \beta \right. \\ & \left. + \frac{1}{2} \epsilon_0 e^{\beta - \phi} F(\beta, z) \partial_\mu z \partial^\mu z \right]. \end{aligned} \quad (4.29)$$

Here, the function F is as defined in Eq. (4.25).

It is interesting to compare this four-dimensional effective action to its counterpart (2.13) obtained in the M-theory case. Obviously, the only difference arises in the kinetic term for z where the function F appears in Eq. (4.29) but not in the M-theory result (2.13). A detailed comparison requires computing this function from Eq. (4.25) by inserting an explicit potential U . However, the qualitative features of F can be easily read off from the alternative expression (4.26). It states that F is the difference of the superpotential on the two boundaries in units of σ and, hence, it is simply the “charge difference” between the two boundaries. Suppose, that the kink’s core is well within the physical space and away from the boundaries, so that $z \in [0, 1]$ and sufficiently different from the boundary values 0, 1. The field χ will then be very close to the minimum $\chi = nv$ at the $y=0$ boundary and very close to the minimum $\chi = (n+1)v$ at the other boundary. The charge difference between the boundaries and, hence, the function F , is, therefore, very close to one. If, on the other hand, the virtual position of the kink’s core is at $z > 1$ ($z < 0$) and sufficiently away from the boundary, χ will be close to the minimum $\chi = nv$ [$\chi = (n+1)v$] on *both* bound-

aries. Hence the function F is approximately zero in this case. This obviously implies a nontrivial behavior of F close to the boundaries for $z \approx 0$ and $z \approx 1$. As a result, for the kink being inside the physical space and away from the boundaries by a distance large compared to its width the effective action (4.29) completely agrees² with the M-theory result (2.13). Conversely, if the kink approaches one of the boundaries or collides with it, that is, $z \rightarrow 0, 1$, the function F becomes nontrivial and the effective theories (4.29) and (2.13) differ substantially. It is clear then, that the effective theory (4.29) carries some memory of the presence of the boundaries while the M-theory action (2.13) does not. For this reason, studying the collision process in the context of Eq. (4.29) is an interesting problem which we will address in Sec. VI.

V. AN EXPLICIT EXAMPLE

In this section, we consider the explicit example of the double-well potential

$$U = m^2 (v^2 - \chi^2)^2, \quad (5.1)$$

where m is a constant. As it stands this potential does, of course, not satisfy our periodicity requirement for U . However, for our purposes this is largely irrelevant since the single-kink solution in which we are interested here probes the potential only between the two minima.³ The associated superpotential is given by

$$w = m\chi \left(v^2 - \frac{1}{3} \chi^2 \right). \quad (5.2)$$

Hence the elementary charge unit σ and ϵ_0 take the form

$$\sigma = w(v) - w(-v) = \frac{4}{3} m v^3,$$

$$\epsilon_0 = \pi \rho \sigma = \frac{4}{3} \pi \rho m v^3. \quad (5.3)$$

The kink-solution for this potential is of the general form (4.1)–(4.4) with the functions C and f given by

$$C(x) = v \tanh(x) \quad (5.4)$$

and

$$f = \frac{1}{\epsilon_0 e^{\beta}} \left[c - \frac{1}{3} v^2 \tanh^2 \xi - \frac{4}{3} \ln(\cosh \xi) \right],$$

²We recall that our kink carries a single charge and we should, therefore, set $\beta_3 = 1$ in Eq. (2.15) to obtain perfect agreement.

³One way to satisfy all earlier requirements is to restrict the potential (5.1) to the interval $[-v, v]$ and continue it periodically outside. The subsequent results do not depend on whether one works with this periodic version of the potential or simply with its original form (5.1).

$$\xi = \frac{e^\beta}{\mu} \left(\frac{y}{\pi\rho} - z \right), \quad (5.5)$$

where the thickness μ of the kink can be identified as

$$\mu = \frac{1}{mv\pi\rho}. \quad (5.6)$$

The constant c in Eq. (5.5) has to be fixed so that $\langle f \rangle = 0$, as discussed before. This leads to an expression involving dilogarithms and we will not carry this out explicitly.

Instead, we consider the limit where the strong-coupling expansion parameter ϵ remains small, so that f becomes irrelevant and our general result (4.18)–(4.21) holds. The functions J_n can now be explicitly computed inserting the potential (5.1) and (5.4) into their definition (4.22). This leads to

$$J_n(x) = \frac{3}{4} \int_0^x d\tilde{x} \frac{\tilde{x}^n}{\cosh^4 \tilde{x}}. \quad (5.7)$$

This, together with Eqs. (4.18)–(4.21) completely determines the moduli-space metric for the double-well potential as long as $\epsilon < 1$. While the above integrals can be carried out for all relevant values $n=0,1,2$, the cases $n=1$ and $n=2$ lead to somewhat complicated expressions, the latter involving a dilogarithm. However, J_0 takes the relatively simple form

$$J_0(x) = \frac{1}{2} \tanh x + \frac{\sinh x}{4 \cosh^3 x}. \quad (5.8)$$

As is clear from the general case discussed in the previous section, for a kink with small width, that is, $e^{-\beta}\mu < 1$, fortunately J_0 is the only relevant function. In this limit, the moduli-space metric is, therefore, given by the general form (4.28) which we repeat for convenience

$$G = \text{diag} \left(\frac{1}{2}, \frac{3}{2}, \epsilon_0 e^{\beta-\phi} F(\beta, z) \right). \quad (5.9)$$

The function F , defined in Eq. (4.25), now takes the explicit form

$$F(\beta, z) = J_0[e^\beta \mu^{-1}(1-z)] - J_0(-e^\beta \mu^{-1}z), \quad (5.10)$$

where J_0 is given in Eq. (5.8). Inserting this result into Eq. (4.29) completely determines the four-dimensional kink effective theory for $\epsilon < 1$ and $e^{-\beta}\mu < 1$. The function F above indeed has the properties mentioned in the previous section, namely $F \approx 1$ for z well inside the interval $[0,1]$ and $F \rightarrow 0$ for $z \rightarrow \pm\infty$. The typical shape of F as a function of z is shown in Fig. 2.

VI. KINK EVOLUTION EQUATIONS

We will now study the time-evolution of the kink based on the effective four-dimensional action derived in the previous section. The collision of the kink with one of the

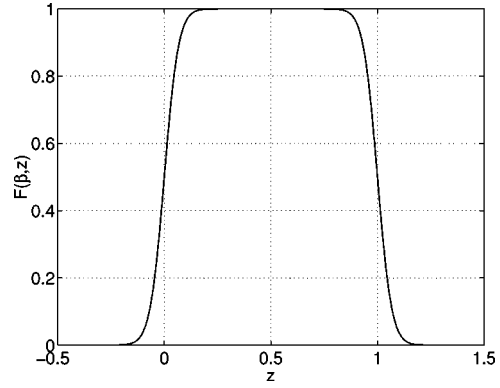


FIG. 2. The function F which enters the effective four-dimensional action of the kink as a function of z for $e^\beta \mu^{-1} = 10$.

boundaries will, of course, be of particular interest.

We focus on simple time-dependent backgrounds and a metric of Friedmann-Robertson-Walker form with flat spatial sections, that is

$$ds_4^2 = -dt^2 + e^{2\alpha(t)} d\mathbf{x}^2, \quad (6.1)$$

$$\phi^I = \phi^I(t), \quad (6.2)$$

where $(\phi^I) = (\phi, \beta, z)$. Let us first review the general structure of the evolution equations for backgrounds of this form. From the general sigma-model action (4.11) one obtains the equations of motion

$$3\dot{\alpha}^2 = \frac{1}{2} G_{IJ} \dot{\phi}^I \dot{\phi}^J, \quad (6.3)$$

$$2\ddot{\alpha} + 3\dot{\alpha}^2 = -\frac{1}{2} G_{IJ} \phi^I \phi^J, \quad (6.4)$$

$$\ddot{\phi}^I + 3\dot{\alpha}\dot{\phi}^I + \Gamma_{JK}^I \dot{\phi}^J \dot{\phi}^K = 0, \quad (6.5)$$

where Γ_{JK}^I is the Christoffel connection associated to the sigma-model metric G_{IJ} and the dot denotes the derivative with respect to time. Adding the first two equations, we obtain an equation for the scale factor α alone which can be immediately integrated. Discarding trivial integration constants one finds

$$\alpha = \frac{1}{3} \ln|t|. \quad (6.6)$$

This power-law evolution with power 1/3 is as expected for a universe driven by kinetic energy only. We also remark that we have, as usual, a $(-)$ branch, $t < 0$, with decreasing α and a future curvature singularity at $t=0$ and a $(+)$ branch, $t > 0$, with increasing α and a past curvature singularity at $t=0$. Our subsequent results will apply to both branches although, for the concrete discussion, we will mostly focus on the positive-time branch, where the universe expands. We find it convenient to use the scale factor α , rather than t , as the time parameter in the following. The remaining evolution equations can then be written in the form

$$\phi^{I'''} + \Gamma_{JK}^I \phi^{J'} \phi^{K'} = 0, \quad (6.7)$$

$$\frac{1}{2} G_{IJ} \phi^{I'} \phi^{J'} = 3, \quad (6.8)$$

where the prime denotes the derivative with respect to α . Hence, the scalar fields ϕ^I , viewed as functions of the scale factor α , move along geodesics in moduli space, with initial conditions subject to the constraint (6.8).

Let us now apply these equations to the moduli space metric for the kink in a double-well potential, as computed in the previous section. To keep the formalism as simple as possible we will focus on the case of a small kink width, that is, $e^{-\beta}\mu \ll 1$. The moduli-space metric is then specified by Eqs. (5.9), (5.10) and (5.8). Inserting this metric into Eq. (6.7) we find

$$\phi'' + \epsilon_0 e^{\beta-\phi} F z'^2 = 0, \quad (6.9)$$

$$\beta'' - \frac{1}{3} \epsilon_0 e^{\beta-\phi} F (1 + e^{\beta}\mu^{-1}K) z'^2 = 0, \quad (6.10)$$

$$z'' + (\beta' - \phi')z' + e^{\beta}\mu^{-1}K\beta'z' - \frac{1}{2}e^{\beta}\mu^{-1}Lz'^2 = 0, \quad (6.11)$$

while the constraint (6.8) turns into

$$\frac{1}{4}\phi'^2 + \frac{3}{4}\beta'^2 + \frac{1}{2}\epsilon_0 e^{\beta-\phi} F z'^2 = 3. \quad (6.12)$$

The functions $K=K(\beta, z)$ and $L=L(\beta, z)$ are related to derivatives of $F=F(\beta, z)$ and can be defined in terms of J_0 , Eq. (5.8), as follows:

$$F(\beta, z) = J_0[e^{\beta}\mu^{-1}(1-z)] - J_0(-e^{\beta}\mu^{-1}z), \quad (6.13)$$

$$K(\beta, z) = \frac{(1-z)J'_0[e^{\beta}\mu^{-1}(1-z)] + zJ'_0(-e^{\beta}\mu^{-1}z)}{F(\beta, z)}, \quad (6.14)$$

$$L(\beta, z) = \frac{J'_0[e^{\beta}\mu^{-1}(1-z)] - J'_0(-e^{\beta}\mu^{-1}z)}{F(\beta, z)}. \quad (6.15)$$

The typical shape of F has been indicated in Fig. 2. Figure 3 shows the shape of K and L as a function of z .

The equations of motion are generally quite complicated due to these functions. However, as the figures show F , K and L are nontrivial only in small regions around the boundaries with size set by $\mu e^{-\beta}$ (the width of the kink relative to the orbifold size) while they are relatively simple outside these critical regions. It is, therefore, useful to discuss the asymptotic form of the equations of motion away from the boundaries. First of all, for $z \in [0, 1]$ and away from the boundaries we have

$$F \approx 1, \quad K \approx 0, \quad L \approx 0. \quad (6.16)$$

Hence, for the kink being well inside the physical space the equations of motion (6.9)–(6.12) greatly simplify and be-

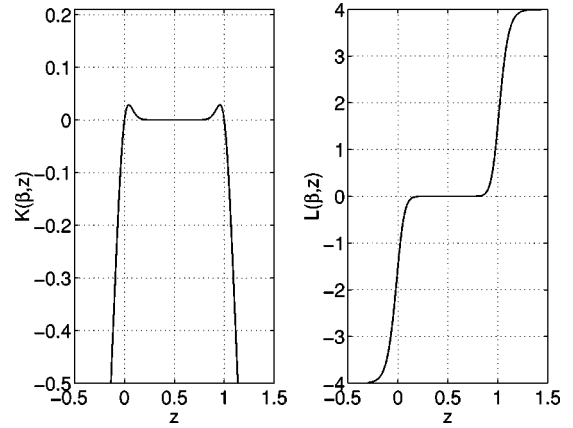


FIG. 3. The functions K and L which enter the effective equations of motion for the kink as a function of z for $e^{\beta}\mu^{-1}=10$.

come, in fact, identical to the analogous equations derived from the M-theory action (2.13).

On the other hand, for $z < 0$ and away from the boundary we have

$$F \approx 0, \quad K \approx 4z, \quad L \approx -4, \quad (6.17)$$

There are analogous results for $z > 1$ but we will focus on the case $z < 0$ for concreteness. Inserting these asymptotic expressions, we see that Eqs. (6.9), (6.10) and (6.12) for ϕ and β decouple from the z equation. They become, in fact, the equations for freely rolling radii and can be easily integrated to give

$$\phi = 3p_{\phi}\alpha + \phi_0, \quad \beta = 3p_{\beta}\alpha + \beta_0 \quad (6.18)$$

where ϕ_0 and β_0 are arbitrary constants and the expansion powers p_{ϕ} and p_{β} satisfy the constraint

$$p_{\phi}^2 + 3p_{\beta}^2 = \frac{4}{3} \quad (6.19)$$

which follows from Eq. (6.12). The evolution of the kink can now be studied in the background of these freely rolling radii. Inserting the above solutions for ϕ and β into the equation for z , Eq. (6.11), we find

$$z'' + 3\delta z' + 2\mu_0^{-1}e^{3p_{\beta}\alpha}(6p_{\beta}z + z')z' = 0, \quad (6.20)$$

where

$$\mu_0 = \frac{\mu}{e^{\beta_0}} \quad (6.21)$$

is the width of the kink relative to the orbifold size initially at $\alpha=0$ and

$$\delta = p_{\beta} - p_{\phi}. \quad (6.22)$$

Hence, for $z < 0$ and away from the boundary the evolution of the kink is described by the single differential equation (6.20).

VII. KINK DYNAMICS AND KINK-BOUNDARY COLLISION

We should now study the solutions to the system (6.9)–(6.12). Given that our main interest is in the collision of the kink with a boundary, ideally, we would like to find solutions with $z \in [0,1]$ initially which evolve towards $z \rightarrow 0$. Given the complexity of the equations, we cannot possibly hope to achieve this analytically. Later, we will address this problem numerically. However, some progress can be made analytically as long as z is away from the boundaries by using the approximate equations for $z \in [0,1]$ or $z < 0$ discussed in the previous section. One may hope that finding such analytical solutions for the evolution up to shortly before and after the collision will lead to a correct qualitative picture of the collision process, roughly by gluing together these two types of solutions across the critical boundary region. As we will see in our numerical analysis, this is indeed the case.

Let us start by looking at the case $z \in [0,1]$. As discussed above, as long as z is not too close to one of the boundaries, the equations of motion reduce to the ones obtained from the M-theory effective action (2.13). Their solutions have been found in Ref. [27] and are explicitly given by

$$\phi = 3p_{\phi,i}\alpha + 3(p_{\phi,f} - p_{\phi,i})\ln(1 + e^{-3\delta_i\alpha})^{-1/3\delta_i} + \phi_0, \quad (7.1)$$

$$\beta = 3p_{\beta,i}\alpha + 3(p_{\beta,f} - p_{\beta,i})\ln(1 + e^{-3\delta_i\alpha})^{-1/3\delta_i} + \beta_0, \quad (7.2)$$

$$z = \frac{d}{1 + e^{3\delta_i\alpha}} + z_0. \quad (7.3)$$

Asymptotically, for $\alpha \rightarrow \pm\infty$, these solutions approach freely rolling radii solutions for ϕ and β while z becomes constant. The early (late) rolling radii solution is characterized by the expansion powers $p_{\phi,i}$ and $p_{\beta,i}$ ($p_{\phi,f}$ and $p_{\beta,f}$). Both sets of expansion powers are subject to the constraint

$$p_{\phi,n}^2 + 3p_{\beta,n}^2 = \frac{4}{3} \quad (7.4)$$

where $n=i,f$ and are related by the linear map

$$\begin{pmatrix} p_{\beta,f} \\ p_{\phi,f} \end{pmatrix} = P \begin{pmatrix} p_{\beta,i} \\ p_{\phi,i} \end{pmatrix}, \quad P = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}. \quad (7.5)$$

Further, we have defined the quantity

$$\delta_i = p_{\beta,i} - p_{\phi,i} \quad (7.6)$$

which can be restricted, without loss of generality, to

$$\begin{aligned} \delta_i > 0 & \quad (-) \text{ branch,} \\ \delta_i < 0 & \quad (+) \text{ branch.} \end{aligned} \quad (7.7)$$

We remark that δ_f , the analogous quantity at late times, is given by

$$\delta_f = p_{\beta,f} - p_{\phi,f} = -\delta_i \quad (7.8)$$

as follows from the map (7.5). The remaining integration constants ϕ_0 , β_0 , z_0 and d are subject to the restriction

$$\phi_0 - \beta_0 = \ln\left(\frac{2\epsilon_0 d^2}{3}\right). \quad (7.9)$$

Note that z_0 specifies the initial position of z which moves by a finite coordinate distance d to its final position $z_0 + d$.

What is the relevance of these solutions in our context? First, we remind the reader that the above solutions play a double-role as exact solution to the M-theory effective action (2.13) and approximate solutions to the kink effective theory if $z \in [0,1]$ and away from the boundaries. In their former role they present another indication that the effective M-theory action (2.13), as it stands, is not adequate to describe the collision process since the boundary values $z = 0,1$ are in no way singled out. In other words, z , as described by these solutions, passes through the boundary without being effected at all. For this reason, they will also be very useful for comparison with solutions to the kink evolution equations, to explicitly see the boundary effect in the latter. In their role as approximate solutions to the kink evolution equations for $z \in [0,1]$ they tell us that the collision can be arranged or avoided depending on a choice of initial conditions. Indeed, the initial position z_0 of the kink and the coordinate distance d by which it moves can be chosen arbitrarily. Hence, for the choice $z_0 \in [0,1]$ and $z_0 + d \in [0,1]$ (and both values away from the boundaries) the entire evolution of the kink is described by the solutions above and a collision with the boundary never occurs. There is, however, a caveat to this argument. While the kink becomes static asymptotically also the strong-coupling expansion parameter ϵ necessarily diverges [27], as can be seen from the above solutions. Therefore, we eventually lose control of our approximation and the effective theory breaks down. Clearly, from the arguments so far, we cannot guarantee that the kink remains static when this happens. In this paper, we will not attempt to improve on this, for example by going back to the five-dimensional theory. Instead, we will be content with arranging a certain characteristic behavior, such as the kink becoming static, to occur for some intermediate period of time before we lose control over the effective theory.

Let us now analyze the evolution of the kink for $z < 0$ and away from the boundary (the case $z > 1$ is similar, of course). In this case, the system is adequately described by the single approximate equation (6.20) for z while ϕ and β are decoupled and evolve according to one of the rolling radii solutions (6.18). Unfortunately, we did not succeed in integrating the z equation in general. However, we can find a number of partial solutions which, as we will see, provide a good indication of the various, qualitatively different types of z evolution.

Let us consider the evolution of z in the background of a special rolling radii solution with a static orbifold, that is,

$$p_\beta = 0, \quad p_\phi = \pm \frac{2}{\sqrt{3}} \quad (7.10)$$

where the two possible values of p_ϕ follow from Eq. (6.19). Equation (6.20) for z then simplifies to

$$z'' - 3p_\phi z' + 2\mu_0^{-1} z'^2 = 0. \quad (7.11)$$

The general solution to this equation can be readily found to be

$$z = z_0 + \frac{v_c}{3p_\phi} \ln \left[1 + \frac{v_0}{v_c} (e^{3p_\phi \alpha} - 1) \right], \quad (7.12)$$

where z_0 and v_0 are integration constants specifying the initial position and velocity of z at $\alpha=0$, that is, $z_0 = z(\alpha=0)$ and $v_0 = z'(\alpha=0)$. Here, we are interested in solutions where z_0 is negative and as close to the boundary as is compatible with the validity of Eq. (7.11). In addition, we need $v_0 < 0$ so z evolves into the region well-approximated by Eq. (7.11). The parameter v_c is defined as

$$v_c = \frac{3}{2} p_\phi \mu_0. \quad (7.13)$$

Let us discuss the properties of this solution for an expanding universe starting with the case $p_\phi = +2/\sqrt{3}$. It is easy to see from Eq. (7.12) that, independent of the initial velocity v_0 , z always diverges to $-\infty$ at some finite value of the scale factor α , in this case. For $p_\phi = -2/\sqrt{3}$, however, the situation is somewhat more complicated and depends on the relation between $|v_0|$ and $|v_c|$. One has to distinguish the following three cases:

- (i) $|v_0| < |v_c|$: z converges exponentially to a constant.
- (ii) $|v_0| = |v_c|$: z diverges to $-\infty$ as $\alpha \rightarrow \infty$.
- (iii) $|v_0| > |v_c|$: z diverges to $-\infty$ at a finite value of α .

Hence, we see that v_c plays the role of a critical velocity. As we will confirm later, these three cases already represent the three types of qualitatively different behavior which can be observed for the full z equation (6.20) or even the complete system (6.9)–(6.12).

We should remark, though, that the second case $|v_0| = |v_c|$ while typical in that z diverges as $\alpha \rightarrow \infty$ is not representative as far as the nature of the divergence is concerned. While its divergence is linear in α , the more characteristic case is an exponential divergence in α . The existence of such exponential divergences can be seen from the special solution

$$z = \frac{\mu_0 p_\phi}{2p_\beta} e^{-3p_\beta \alpha} \quad (7.14)$$

to Eqs. (6.20). While this represents an exact solution for all values of p_ϕ and p_β we have to restrict signs to $p_\beta < 0$ and $p_\phi > 0$ so that z is negative and moves towards $-\infty$. Within this range of p_ϕ and p_β , however, the above solution shows an exponential divergence of z as $\alpha \rightarrow \infty$.

After having identified the qualitatively different types of z evolution we can now ask more systematically, based on the z equation (6.20), which type is realized for a given set of parameters and initial conditions. As can be seen from a

rescaling of z in Eq. (6.20) the type of evolution cannot depend on the value of μ_0 . The only possible dependence is, therefore, on p_β [recall that, for given p_β , p_ϕ is determined, up to a sign, from Eq. (6.19)] and the initial velocity $v_0 = z'(\alpha=0)$. A relevant question in this context concerns the stability of the solution $z = \text{const}$ which can be viewed as the limit of the exponentially converging case 1. Writing

$$z = z_0 + \zeta(\alpha), \quad (7.15)$$

where $z_0 < 0$, the linearized evolution equation for ζ is, from Eq. (6.20), given by

$$\zeta'' = -3[\delta + 4\mu_0^{-1} z_0 p_\beta e^{3p_\beta \alpha}] \zeta'. \quad (7.16)$$

We conclude that the solution $z = \text{const}$ can only be stable if

$$p_\beta < 0 \quad \text{and} \quad \delta = p_\beta - p_\phi > 0. \quad (7.17)$$

It is only then that we expect the first case of convergent z to be realized.

This can indeed be verified by a numerical integration of Eq. (6.20). Solutions with converging z exist if and only if the conditions (7.17) are satisfied and, in addition, if the initial velocity $|v_0|$ is smaller than a certain critical velocity v_c . A simple scaling argument shows that

$$v_c = h(p_\beta, p_\phi) \mu_0 \quad (7.18)$$

where h is a function which, from the numerical results, turns out to be of $\mathcal{O}(1)$ and slowly varying. What happens outside the region (7.17)? If we leave this range by crossing $p_\beta = 0$ we find for small positive p_β and $|v_0|$ below the critical velocity that z still converges at first but then, in accordance with our analytic argument, develops an instability, which drives it to $-\infty$ at finite α . The intermediate stable phase gradually disappears as one increases p_β . For $p_\beta > 0$ and $|v_0|$ above the critical velocity one always finds divergence to $-\infty$ at finite α . Hence, for $p_\beta > 0$ we are always in the third case above. As we leave the region (7.17) crossing $\delta = 0$ we find case (ii) is realized below and case (iii) above the critical velocity. However, as δ becomes more negative, the critical velocity decreases rapidly until we are left with case (iii) only.

In summary, the converging case (i) is only found in the range (7.17) and for initial velocities smaller than a certain critical value while otherwise z always diverges to $-\infty$ typically according to case (iii) at finite scale factor α .

We can now try to combine the information we have gathered about the evolution of the system before and after the collision to set up criteria which will allow us to decide the outcome of a collision process. Let us consider a particular solution (7.1)–(7.3) for the evolution inside the interval $z \in [0, 1]$. As we have already mentioned, the distance by which the kink moves is a free parameter so a collision may never occur. Then, this solution describes the full evolution of the system as far as it is accessible within the four-dimensional effective theory. On the other hand, if initial

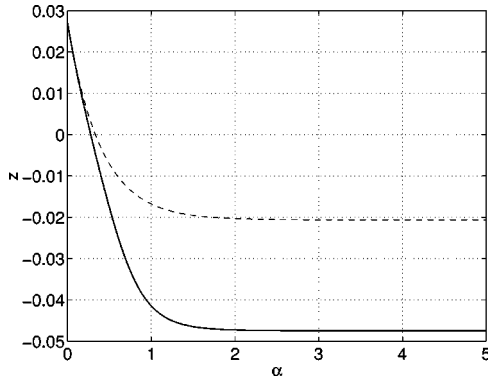


FIG. 4. Position modulus z for the kink (solid line) and M-theory three-brane (dashed line) as a function of the scale factor α . The initial conditions have been chosen as $z_{\text{col}}=0.027$, $z'_{\text{col}}=-0.12$, $\beta_{\text{col}}=2.0$, $\beta'_{\text{col}}=-0.72$, $\phi_{\text{col}}=16.14$, $\phi'_{\text{col}}=-3.23$.

conditions are chosen so that a collision does occur, the particular solution (7.1)–(7.3) will determine the velocities z'_{col} , ϕ'_{col} and β'_{col} right before the collision. We can then, approximately, identify $v_0 \approx z'_{\text{col}}$, $3p_\phi \approx \phi'_{\text{col}}$ and $3p_\beta \approx \beta'_{\text{col}}$ and apply the previous results for the evolution at $z < 0$. One concludes that only for a very low-impact collision with small z'_{col} and an orbifold size which, at collision, decreases less rapidly than the dilaton, that is $\beta'_{\text{col}} < 0$ and $\phi'_{\text{col}} - \beta'_{\text{col}} > 0$, does z converge to a constant. Otherwise z diverges to $-\infty$ and this can, in fact, be viewed as the generic case.

Of course, the criteria above may be somewhat inaccurate since we have ignored the complicated structure of the evolution equations near the boundary. We have, therefore, numerically integrated the full system (6.9)–(6.12) to test the above criteria for the outcome of a collision process. It turns out that, in broad terms, the picture remains qualitatively the same.

Starting with z near zero inside the $[0,1]$ interval, we went around the ellipse $(\phi'_{\text{col}})^2 + 3(\beta'_{\text{col}})^2 \approx 12$. Note that in this case the exact identity cannot be observed since the constraint equation (6.12) includes an extra term proportional to $(z'_{\text{col}})^2$. Nevertheless the correction is always small since we set $\phi_{\text{col}} - \beta_{\text{col}}$ to a large negative value. This makes the initial value for ϵ very small and allows us, for the cases where ϵ grows, to follow the evolution for longer times until $\epsilon \approx 1$ and the four-dimensional effective theory breaks down. We also chose a large initial β_{col} so that $e^{-\beta}\mu$ remains as small as possible during the evolution, for the cases with $\beta'_{\text{col}} < 0$. In all cases we set $\epsilon_0 = 1$ and $\mu = 0.2$.

For each of these sets of initial conditions we then varied z'_{col} from zero upwards and looked for changes in the large time behavior of z . The numerical results were obtained by evolving Eqs. (6.9)–(6.11) using a fourth-order fixed step Runge-Kutta method. The accuracy of the method was checked by confirming that the constraint equation Eq. (6.12) was satisfied throughout the evolution. The individual terms on the left hand side of Eq. (6.12) should sum to 3, and typically after 2000 time-steps of size 0.01 the deviation from this value was smaller than 0.01%. In the worst cases, where the equations of motion are no longer valid because

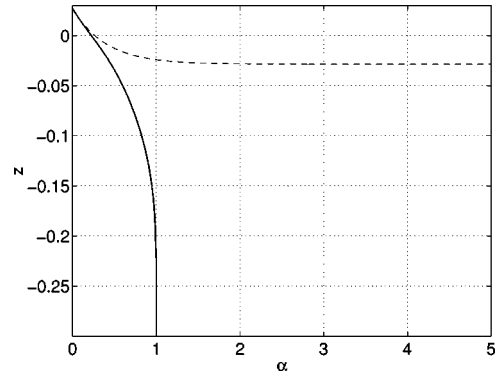


FIG. 5. Same as in Fig. 4 but with $z'_{\text{col}} = -0.14$.

one of the assumptions has broken down, the sum never gets above 0.2%.

In Fig. 4 we have an example of the first type of behavior, for a small negative value of β'_{col} . After crossing the boundary at $z=0$ the kink relaxes to a stable constant solution. For early times this solution matches the one obtained from the M-theory effective action for the same initial conditions. Nevertheless, as soon as the kink approaches the boundary the two start differing, converging to different asymptotic values.

For a slightly higher value of the initial velocity the difference is even more striking, as shown in Fig. 5. In this case z diverges in finite time, indicating that we are above the critical velocity. This third case turns out to be the most common, as already observed in the simplified system. Only for $\beta'_{\text{col}} < 0$ and $\beta'_{\text{col}} - \phi'_{\text{col}} > 0$ and z'_{col} below the critical velocity does the system avoid singular behavior.

In Fig. 6 we have an example for a solution corresponding to case 2. Here both β'_{col} and ϕ'_{col} are negative and we are below the critical velocity. As a consequence of $\delta \approx \beta'_{\text{col}} - \phi'_{\text{col}} < 0$, z does not relax to a constant but its magnitude increases exponentially instead. In this case the solution has to be taken with care, since $e^{-\beta}\mu$ quickly becomes large in the exponential regime and the equations of motion stop providing a reliable approximation.

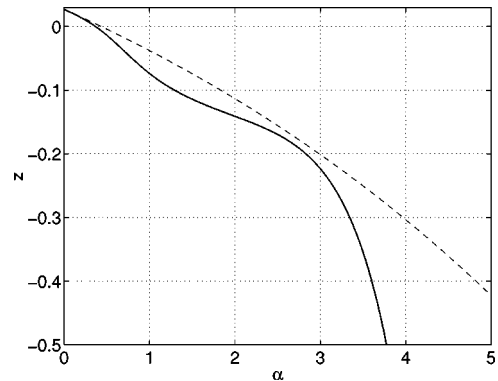


FIG. 6. Position modulus z for the kink (solid line) and M-theory three-brane (dashed line) as a function of the scale factor α . The initial conditions have been chosen as $z_{\text{col}}=0.027$, $z'_{\text{col}}=-0.060$, $\beta_{\text{col}}=2.0$, $\beta'_{\text{col}}=-1.77$, $\phi_{\text{col}}=14.75$, $\phi'_{\text{col}}=-1.61$.

Finally we have checked that once we go above the critical velocity, z always diverges for finite α . It is well known that in ϕ^4 theory when a kink-antikink collision takes place, above a certain limit velocity, they reflect and bounce back [31] (for lower velocities they can either reflect or form a bound state). This behavior is a consequence of a resonance effect between the kink pair and higher field modes, so we should not be surprised to observe it in the context of our four-dimensional effective action. This does not yet exclude the possibility of a bounce in a high-velocity regime which is accessible only in the context of the full five dimensional theory, a question which is currently under investigation [32].

What do these results imply in terms of the five-dimensional defect model (3.1)? As we have seen, if z starts its evolution within the interval $[0,1]$ and subsequently collides with a boundary (at $z=0$) it is generically driven to $-\infty$ very rapidly. It should be stressed that the z kinetic energy remains finite at this singularity. Nevertheless, we do expect the effective four-dimensional theory to break down eventually, as $z \rightarrow -\infty$. This is because some of the higher-order terms we have neglected are likely to grow with z , in a way similar to the linear z term in Eq. (6.20). However, at least for sufficiently small expansion parameters ϵ and $\mu e^{-\beta}$ the four-dimensional theory will be valid some way into the singularity. Hence, we can conclude that a five-dimensional kink, interpolating between the vacua $\chi=nv$ and $\chi=(n+1)v$ which collides with the boundary at $z=0$ effectively disappears and leaves the field χ in the vacuum state $\chi=(n+1)v$ (and an analogous statement holds for collision with the boundary at $z=1$). From the M-theory perspective, such a process corresponds to a transition

$$\begin{aligned} (\beta_1, \beta_2, \beta_3) &= (n, -(n+1), 1) \rightarrow (\beta_1, \beta_2, \beta_3) \\ &= (n+1, -(n+1), 0) \end{aligned} \quad (7.19)$$

between two different sets of charges and, hence, topologically different compactifications.

VIII. CONCLUSION AND OUTLOOK

In this paper, we have presented a toy defect model for five-dimensional heterotic brane-world theories, where three-branes are modeled by kink solutions of a bulk scalar field χ . We have shown that the vacuum states of this defect model correspond to a class of topologically distinct M-theory models characterized by the charges β_1 and β_2 on the boundaries and the three-brane charge β_3 . Specifically, we have seen that a state where χ equals one of the minima $\chi=\chi_n=nv$ of the potential, where $n \in \mathbb{Z}$, corresponds to a state with charges $(\beta_1, \beta_2, \beta_3) = (n, -n, 0)$, that is, an M-theory model without three-branes. If, on the other hand, χ represents a kink solution interpolating between the minima $\chi=nv$ and

$\chi=(n+1)v$ the associated M-theory charges are $(\beta_1, \beta_2, \beta_3) = (n, -(n+1), 1)$ corresponding to a model with a single-charged three-brane.

We have computed the effective four-dimensional action associated to a kink solution and have studied the time-evolution of a kink in this context. Our results show that, generically, a collision of the kink with a boundary will lead to a transition between the two types of vacua mentioned above. In other words, the kink will disappear after collision which corresponds to a transition between a state with a single-charged three brane and a state without a three-brane. We should stress that these results can only be trusted as far as the four-dimensional effective action is valid. In particular, we cannot exclude the possibility of the kink bouncing off the boundary for higher collision velocities. A future full 5D study of the problem will answer this question and clarify the nature of the collision process in general.

There are several interesting directions which may be pursued on the basis of these results. Clearly, our original M-theory model as well as the associated defect model are rather simple and a number of possible extension and modifications come to mind. First of all, we may try to modify our defect model by including more than one additional bulk scalar field, in particular to allow for exact BPS multikink solutions. One may ask whether the defect model can be embedded into a five-dimensional $N=1$ supergravity theory as is the case for the original M-theory model. Further, there are a number of generalizations of five-dimensional heterotic M theory, such as including a more general set of moduli fields [8], which one may try to implement into the defect model. For example, including the general set of Kahler moduli would allow one to study topological transitions of the underlying Calabi-Yau space through flops, in addition to the types of topology change considered in this paper.

Perhaps the most interesting direction is to study the evolution of more complicated configurations of our defect model (3.1). For example, one could envisage evolving the field χ from some initial (say thermal) distribution to see which type of brane-network develops at late time [32]. In particular, one would like to answer the important question whether the system can evolve from a brane-gas to a brane-world state. If this is indeed what happens such an approach will lead to predictions for the late-time brane-world that has evolved, given a certain class of plausible initial states. Concretely, within the context of the simple model presented in this paper, we may expect predictions for the charges β_i in this case. As we have discussed, the values of these charges are correlated with important properties of the theory such as the type of gauge group. Optimistically, we may therefore hope that our approach leads to prediction for such important low-energy data, at least within a restricted class of associated M-theory compactifications.

ACKNOWLEDGMENTS

A.L. and N.D.A. are supported by the PPARC.

- [1] P. Horava and E. Witten, Nucl. Phys. **B475**, 94 (1996).
- [2] E. Witten, Nucl. Phys. **B471**, 135 (1996).
- [3] P. Horava, Phys. Rev. D **54**, 7561 (1996).
- [4] A. Lukas, B.A. Ovrut, and D. Waldram, Nucl. Phys. **B532**, 43 (1998).
- [5] A. Lukas, B.A. Ovrut, and D. Waldram, Phys. Rev. D **59**, 106005 (1999).
- [6] A. Lukas, B.A. Ovrut, K.S. Stelle, and D. Waldram, Phys. Rev. D **59**, 086001 (1999).
- [7] J.R. Ellis, Z. Lalak, S. Pokorski, and W. Pokorski, Nucl. Phys. **B540**, 149 (1999).
- [8] A. Lukas, B.A. Ovrut, K.S. Stelle, and D. Waldram, Nucl. Phys. **B552**, 246 (1999).
- [9] B. Andreas, J. High Energy Phys. **01**, 011 (1999).
- [10] G. Curio, Phys. Lett. B **435**, 39 (1998).
- [11] R. Donagi, A. Lukas, B.A. Ovrut, and D. Waldram, J. High Energy Phys. **05**, 018 (1999).
- [12] R. Donagi, A. Lukas, B.A. Ovrut, and D. Waldram, J. High Energy Phys. **06**, 034 (1999).
- [13] R. Donagi, B.A. Ovrut, T. Pantev, and D. Waldram, Adv. Theor. Math. Phys. **5**, 93 (2002).
- [14] R. Donagi, B.A. Ovrut, T. Pantev, and D. Waldram, Class. Quantum Grav. **17**, 1049 (2000).
- [15] R. Donagi, B.A. Ovrut, T. Pantev, and D. Waldram, Adv. Theor. Math. Phys. **5**, 499 (2002).
- [16] E. Witten, Nucl. Phys. **B460**, 541 (1996).
- [17] O.J. Ganor and A. Hanany, Nucl. Phys. **B474**, 122 (1996).
- [18] B.A. Ovrut, T. Pantev, and J. Park, J. High Energy Phys. **05**, 045 (2000).
- [19] O. DeWolfe, D.Z. Freedman, S.S. Gubser, and A. Karch, Phys. Rev. D **62**, 046008 (2000).
- [20] A. Sen, Int. J. Mod. Phys. A **14**, 4061 (1999).
- [21] J.A. Harvey and P. Kraus, J. High Energy Phys. **04**, 012 (2000).
- [22] J. Khoury, B.A. Ovrut, P.J. Steinhardt, and N. Turok, Phys. Rev. D **64**, 123522 (2001).
- [23] J. Khoury, B.A. Ovrut, N. Seiberg, P.J. Steinhardt, and N. Turok, Phys. Rev. D **65**, 086007 (2002).
- [24] M. Brandle and A. Lukas, Phys. Rev. D **65**, 064024 (2002).
- [25] J. Derendinger and R. Sauser, Nucl. Phys. **B598**, 87 (2001).
- [26] A. Strominger, Nucl. Phys. **B343**, 167 (1990); **B353**, 565(E) (1991).
- [27] E.J. Copeland, J. Gray, and A. Lukas, Phys. Rev. D **64**, 126003 (2001).
- [28] K. Skenderis and P.K. Townsend, Phys. Lett. B **468**, 46 (1999).
- [29] F. Bonjour, C. Charmousis, and R. Gregory, Phys. Rev. D **62**, 083504 (2000); B. Carter and R. Gregory, *ibid.* **51**, 5839 (1995).
- [30] M.A. Shifman, Phys. Rev. D **57**, 1258 (1998); C. Bachas, J. Hoppe, and B. Pioline, J. High Energy Phys. **07**, 041 (2001); J.P. Gauntlett, D. Tong, and P.K. Townsend, Phys. Rev. D **64**, 025010 (2001); A.A. Izquierdo, M.A. Leon, and J.M. Guilarte, *ibid.* **65**, 085012 (2002); D. Tong, *ibid.* **66**, 025013 (2002).
- [31] D.K. Campbell, J.F. Schonfeld, and C.A. Wingate, Physica D **9**, 1 (1983).
- [32] N. D. Antunes, E. J. Copeland, M. Hindmarsh, and A. Lukas (in preparation).